

NEW CONTRAST ENHANCEMENT TECHNIQUES BASED ON HISTOGRAM EQUALIZATION CONCEPT FOR GRAY SCALE IMAGE

by

LIM SHENG HOONG

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LIST OF ABBREVIATIONS

AAMBE	Average Absolute Mean Brightness Error
AC	Average Contrast
AE	Average Entropy
AMHE	Adaptive Modified Histogram Equalization
AT	Average Time
BBHE	Brightness Preserving Bi-Histogram Equalization
BHE	Bi-Histogram Equalization
BHENM	Bi-Histogram Equalization with Neighborhood Metric
BLS	Black Level Stretch
BO	Bin Overflow
BPDFHE	Brightness Preserving Dynamic Fuzzy Histogram Equalization
BPDHE	Brightness Preserving Dynamic Histogram Equalization
BPPLHE	Brightness Preserving Plateau Limit Histogram Equalization
BU	Bin Underflow
BUBOHE	Bin Underflow and Bin Overflow Histogram Equalization
CHE	Clipped Histogram Equalization
CLAHE	Clipped Limited Adaptive Histogram Equalization
DHE	Dynamic Histogram Equalization
DPPLHE	Detail Preservation Plateau Limit Histogram Equalization
DRBHE	Dynamic Range Bi-Histogram Equalization
DRHE	Dynamic Range Histogram Equalization
DSIHE	Dualistic Sub-Image Histogram Equalization
FHE	Fuzzy Histogram Equalization
GCCHHE	Gain Controllable Clipped Histogram Equalization

GHE	Global Histogram Equalization
HE	Histogram Equalization
HERO	Histogram Equalization with Range Offset
HEwVED	Histogram Equalization with Variable Enhancement Degree
HSV	Human Visual System
IQPLBHE	Improved Quantized Plateau Limits Bi-Histogram Equalization
LHE	Local Histogram Equalization
MHE	Multiple Histogram Equalization
MHEBP	Multipeak Histogram Equalization with Brightness Preservation
MMBEBHE	Minimum Mean Brightness Error Bi-Histogram Equalization
MSSI	Mean Structural Similarity Index
NMHE	Neighborhood Metric Histogram Equalization
OBHE	Object Based Histogram Equalization
PLHE	Plateau Limit Histogram Equalization
PSNR	Peak Signal to Noise Ratio
QPLBHE	Quantized Plateau Limits Bi-Histogram Equalization
RHE	Recursive Histogram Equalization
RMSHE	Recursive Mean Separate Histogram Equalization
RSIHE	Recursive Sub-Image Histogram Equalization
RSWHE	Recursively Separated and Weighted Histogram Equalization
SAPHE	Self-Adaptive Plateau Histogram Equalization
SHE	Single Histogram Equalization
SHMS	Simple Histogram Modification Scheme
WCHE	Weight Clustering Histogram Equalization
WHE	Weighted Histogram Equalization

WLS	White Level Stretch
WMSHE	Weighting Mean-separated Sub-Histogram Equalization
WTHE	Weighted Thresholded Histogram Equalization

LIST OF SYMBOLS

α	Enhancement rate factor
$AMBE$	Absolute Mean Brightness Error
β	Degree of mean brightness and contrast enhancement
C	Cluster
CDF	Cumulative density function
CDF_L	Cumulative density function of the lower sub-histogram
CDF_H	Cumulative density function of the higher sub-histogram
CDF_{NLN}	Normalized new probability density function of the lower sub-histogram
CDF_{NHN}	Normalized new probability density function of the higher sub-histogram
CR	Clipped rate
CW	Cluster width
D_L	Gray level ratio differences of the lower sub-histogram
D_H	Gray level ratio differences of the higher sub-histogram
DR_C	Dynamic range of a cluster
DR_m	Dynamic range of each cluster
ε	Amount of emphasis given on the enhancement rate
ER	Enhancement rate
exp	Exponent
$E(Y)$	Mean of the enhanced image
$E(X)$	Mean of the input image
$factor$	New gray level range deciding factor
G^G	Global gain

G^T	Total gain
G_L^L	Lower local gain
G_H^L	Higher local gain
γ	User variable used to control the enhancement rate of the black level and white level regions
<i>Gaussian</i>	Gaussian filter
<i>GR</i>	Gray level ratio
<i>HIS</i>	Histogram
<i>HIS_C</i>	Clipped histogram
<i>HIS_d</i>	Histogram of the differential operation
<i>HIS_H</i>	Higher sub-histogram
<i>HIS_L</i>	Lower sub-histogram
<i>HIS_M</i>	Modified histogram
I_{height}	Image height
I_{width}	Image width
L	Total number of gray levels
l	Gray level
l_B	<i>SP</i> of the black level region
l_d	Median of the input histogram
l_g	Middle gray level
l_{MIN}	Minimum gray level
l_{MAX}	Maximum gray level
l_μ	Mean gray level
$l_{\mu H}$	Mean gray level of the higher sub-histogram

$l_{\mu L}$	Mean gray level of the lower sub-histogram
l_T	Total number of gray level
l_W	SP of the white level region
\log	Logarithm
M	Total number of images
m	Number of cluster after merging process
MBE	Mean brightness error
μ	Mean
N	Total number of pixels
N_H	Total number of pixels in the higher sub-histogram
n_i	Total number of pixels of i -th sub histogram
n_k	Bins of the k -th gray level
N_L	Total number of pixels in the lower sub-histogram
$Output(x,y)$	Enhanced image
$Output_{norm}(x,y)$	Normalized enhanced image
PDF	Probability density function
PDF_H	Probability density function of the higher sub-histogram
PDF_L	Probability density function of the lower sub-histogram
PDF_{ll}	Lower thresholds
PDF_{MAX}	Highest probability
PDF_{MID}	Mean value of l_{MAX} and l_{MIN}
PDF_{MIN}	Lowest probability
PDF_{New}	New probability density function
PDF_{NH}	New probability density function of the higher sub-histogram
PDF_{NL}	New probability density function of the lower sub-histogram

PDF_s	Smoothed probability density function
PDF_{ul}	Upper thresholds
PDF_W	Weighted probability density function
PDF_{WN}	Weighted and normalized probability density function
Pk_H	Peak bin of the higher sub-histogram
Pk_L	Peak bin of the lower sub-histogram
$PL_{BPPLHE-D}$	Median plateau limit of BPPLHE
$PL_{BPPLHE-M}$	Mean plateau limit of BPPLHE
PL_H	Plateau limit of higher sub-histogram
PL_L	Plateau limit of lower sub-histogram
R	New gray level range
r	Recursion level
RD	Remainder
RT	Runtime
σ	Standard deviation
S_{MAX}^i	Maximum gray level at the i -th sub-histogram
S_{MIN}^i	Minimum gray level at the i -th sub-histogram
$SMBE$	Scaled mean brightness error
SP	Separating point
$span$	Span
SP_H	Separating point of the higher sub-histogram
SP_L	Separating point of the lower sub-histogram
SP_{New}	New separating point
T	Transformation function
TPM	Processing time per megapixels

ν	Upper threshold normalized to PDF_{MAX}
W	Cluster weight
WMV_r^i	Weighting mean value
WR_1	Cluster weight ratio 1
WR_2	Cluster weight ratio 2
x	Coordinate relative to the center of the kernel
$<<$	Left shift operator

TEKNIK PENINGKATAN KONTRAS BAHARU BERASASKAN KONSEP PENYERAGAMAN HISTOGRAM UNTUK IMEJ PARAS KELABU

ABSTRAK

Walaupun penyeragaman histogram “*Histogram Equalization*” (HE) terkenal dengan kemudahan dan keberkesanannya dalam peningkatan kontras imej, namun, ia terdedah kepada masalah perubahan kecerahan berlebihan, ketepuan intensiti dan peningkatan kesan hingar. Secara umumnya, kaedah-kaedah berasaskan HE konvensional dibahagikan kepada dua kategori, iaitu kaedah pengekalan kecerahan (i.e., direka untuk mengekal kecerahan dan meningkatkan kontras imej semulajadi yang diambil di bawah keadaan cahaya biasa) dan kaedah pengekalan perincian (i.e., direka untuk meningkatkan visual dan meningkatkan kecerahan imej semulajadi yang diambil di bawah keadaan kurang cahaya). Dalam kajian ini, satu kaedah pengekalan kecerahan iaitu “*Improved Quantized Plateau Limits Bi-Histogram Equalization*” (IQPLBHE) dan satu kaedah pengekalan perincian iaitu “*Dynamic Range Bi-Histogram Equalization*” (DRBHE) diperkenalkan. Pada asasnya, kaedah IQPLBHE yang dicadangkan memisahkan histogram imej ujian kepada dua sub-histogram terlebih dahulu. Kemudian, had-had dataran dikenalpasti dari setiap sub-histogram yang akan digunakan untuk mengubahsuai sub-histogram tersebut. Penyeragaman histogram kemudian dilaksanakan kepada kedua-dua sub-histogram secara berasingan. Sebaliknya, kaedah DRBHE yang dicadangkan memisahkan histogram imej ujian kepada dua sub-histogram terlebih dahulu. Kemudian, fungsi ketumpatan kebarangkalian baru berdasarkan maklumat tempatan dicadangkan. Seterusnya, ketumpatan terkumpul fungsi normal diaplikasikan. Akhirnya, satu titik pemisahan baharu berdasarkan konsep kaedah “*Dynamic Range Histogram Equalization*” (DRHE) dikira sebelum gabungan HE and DRHE diaplikasikan. Keputusan-

keputusan kualitatif dan kuantitatif analisis menunjukkan bahawa kedua-dua kaedah yang dicadangkan mempunyai prestasi yang baik. Selain itu, kedua-dua kaedah yang dicadangkan mempunyai kelebihan daripada segi algoritma yang ringkas dan tidak memerlukan penentuan parameter secara manual, ini membuatkan kedua-dua kaedah sesuai diaplikasikan dalam produk elektronik pengguna.

NEW CONTRAST ENHANCEMENT TECHNIQUES BASED ON HISTOGRAM EQUALIZATION CONCEPT FOR GRAY SCALE IMAGE

ABSTRACT

Even though the histogram equalization (HE) is well known for its simplicity and effectiveness in image contrast enhancement, nevertheless, it does suffer from excessive brightness change, intensity saturation and noise amplification problems. In general, the conventional HE based methods are divided into two categories, namely brightness preservation (i.e., designed to preserve the brightness and improve the contrast of the natural images taken under normal lighting condition) and detail preservation (i.e., improve the visual and increase the brightness of the natural images taken under low lighting condition) methods. In this study, a brightness preservation method, namely Improved Quantized Plateau Limits Bi-Histogram Equalization (IQPLBHE) and a detail preservation method, namely Dynamic Range Bi-Histogram Equalization (DRBHE), are introduced. Basically, the proposed IQPLBHE method first separates the input histogram into two sub-histograms. Then, the plateau limits are calculated from the respective sub-histograms, which are used to modify those sub-histograms. Lastly, HE is then separately performed on the two sub-histograms. On the other hand, the proposed DRBHE method first separates the input histogram into two sub-histograms. Then, a new probability density function is created based on local information. Next, cumulative density function normalization is applied. Lastly, a new separating point is calculated before the combination of HE and DRHE is applied. Qualitative and quantitative analyses results show that both the proposed methods have good performance. Moreover, both the proposed methods have the advantages of being simple and tuning free, which are suitable to be applied in consumer electronic products.

CHAPTER 1

INTRODUCTION

1.1 Background

An enhancement process can be defined as a process of manipulating the original image so that the resultant image is more suitable than the original image for a specific image enhancement application. The scope of image enhancement application is very wide. It includes gray level and contrast manipulation, noise level reduction, edge crispening and sharpening, interpolation and magnification and pseudocoloring (Williams *et. al.*, 2001). However the scope of this thesis is limited to contrast manipulation or contrast enhancement only.

Contrast enhancement is one of the most interesting and important processes in both human and computer vision fields. The main purpose of contrast enhancement is to improve the interpretability or perception of information in images for human viewers, or to provide better input for other automated image processing techniques.

Commonly, contrast enhancement produces an output image which subjectively looks clearer and crispier with better representation of the subtle detail than the original image by changing the intensity values of the input image. It is an indispensable tool for researchers in a wide variety of fields including (but not limited to) medical imaging, color image processing and underwater image processing. Generally, contrast enhancement can be done by stretching the original narrow dynamic range of an image into its full dynamic range (Ekstrom, 1984). This is based on the assumption that the contrast is proportional to the ratio between the brightest and the darkest pixel intensities contained in an image.

1.2 Digital Image Processing

Mathematically, an image can be expressed as a two dimensional function, $f(x, y)$, where x and y are spatial (plane) coordinates, with the amplitude of f at any pair of coordinates (x, y) . The intensity or gray level of the image at a particular point is defined as the amplitude of f at that particular point. A digital image is formed when all the above mentioned elements (i.e., x , y , and the amplitude values of f) are finite and have discrete quantities. Those elements are also called as the picture elements, image elements, pels or pixels. Among all, pixel is widely used to denote as an element of a digital image and it can be illustrated as in Figure 1.1 (Gonzalez and Woods, 2008). On the other hand, when a digital image is processed by a digital computer, it is called digital image processing.

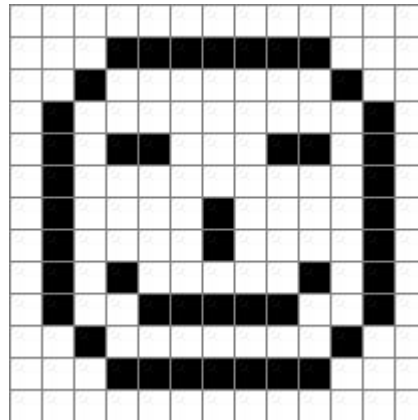


Figure 1.1 A digitized 13×13 black and white image (gray grid lines show the pixels and are not part of the image itself).

Basically, there are three types of digital images available, namely binary images, grayscale images and color images. In binary images, each pixel is represented by a single bit. Since a bit can only represent two values or colors, so every pixel in a binary image must be one of the two colors, usually black or white. One color is used to represent foreground while another color represents the background. Due to the inability to represent intermediate shades of gray, there is a

limit to their usefulness in dealing with photographic images. Figure 1.1 shows an example of binary image.

Grayscale image can be represented as a black and white image which composed exclusively of shades of gray as shown in Figure 1.2. These shades of gray vary from black at the weakest intensity to white at the strongest intensity. Each pixel in the grayscale image holds a single number corresponding to the gray level of the image at a particular location. In addition, grayscale images can also be represented as binary images in a three dimensional space, with the first two dimensions represent the spatial plane, while the third dimension represents brightness (Kansal, 2010). For an 8 bits grayscale image, there are a total of 256 different gray levels can be used to represent each pixel of an image as shown in Figure 1.3. Note that, each pixel uses a single byte (8 bits) of memory.



Figure 1.2 A grayscale image of 'Pepper'.



Figure 1.3 The gray level transition from black to white.

Each pixel in a color image consists of three numbers corresponding to the red, green and blue levels of the image at a particular location as shown in Figure 1.4.

Assuming 256 levels, each color pixel can be stored in three bytes (24 bits) of memory. Note that for images of the same size, a color version will use three times more memory than a grayscale image.



Figure 1.4 A color image of ‘Pepper’.

In this work, only the gray scale images stored as an 8 bits integer are considered and used as input images.

1.3 Contrast Enhancement Techniques

Generally, the contrast enhancement techniques can be divided into three broad categories (Kansal, 2010):

- Spatial domain method
- Frequency domain method
- Fuzzy domain method

For the spatial domain method, the enhancement process is applied directly to the pixels. The spatial domain based contrast enhancement methods are commonly used due to its simplicity, easy to be implemented and produced promising results (Yun et al., 2010; Sim et al., 2010; Ismail and Sim, 2011).

As for frequency domain method, the enhancement process starts by transforming the inputs image from spatial domain to frequency domain. It is then followed by applying the corresponding enhancement before transforming back to spatial domain. The frequency domain based contrast enhancement techniques do not affect the ability to compress the original image and require less memory space as compared to spatial domain techniques (Xia et al., 2011). However, these techniques suffer from block artifacts, extra edges near block boundaries, time consuming, cannot simultaneously enhance all portions of the image very well and selection of the optimum parameters still poses a critical issue for automating the process (Agaian et al., 2007; Xia et al., 2011; Hasikin and Isa, 2012).

For the third category, the fuzzy domain method involves the use of knowledge-base systems that are capable of mimicking the behavior of a human expert. The enhancement process of fuzzy domain is as shown in Figure 1.5 (Hassanien and Badr 2003). Based on previous research, the fuzzy based contrast enhancement techniques allow one to model uncertainty and subject concepts (i.e., grayness ambiguity, geometrical fuzziness and vague knowledge) in a better form than certainty models (Rangasamy et al., 2008; Gopalan et al., 2009; Krishnan and Viswanathan, 2010). However, the performances require complex procedure and long computational time (Hasikin and Isa, 2012).



Figure 1.5 The main principles of fuzzy image enhancement.

In this work, due to its advantages as previously mentioned, the spatial domain method is chosen to be studied. Although the spatial domain method can be further categorized into several categories (i.e., histogram equalization (HE) based (Teuber, 1993), morphology based (Mukhopadhyay and Chanda, 2000), logarithm based (Dargo et al., 2003), intensity pair based (Jen et al., 2005; Kabir et al., 2006, 2009) and standard deviation based (Patrenhalli and Robert, 1981; Diagakis et al., 1993)), this work only studies the HE based method which is well known for its simplicity and effectiveness in image contrast enhancement.

1.4 Histogram Equalization (HE)

Generally, histogram is one of the important features, which is very related to image enhancement. It is because the general overview of an image can be obtained from image statistics contained in the histogram (i.e., mode, median, mean and the dynamic range of an image). Thus, modification of the image's histogram is commonly employed in order to enhance images. One of the well known histogram based digital image techniques is Histogram Equalization (HE). The HE based methods are developed to satisfy human visual system (HVS) where it keens more towards luminance than color (Yun et al., 2010). Moreover, HE has been applied in many fields including medical image processing (Wang et al., 2008; Ziaei et al., 2008; Jagatheeswari et al., 2009; Yousuf and rakib, 2010; Ismail and Sim, 2011), color image processing (Trahalias and Venetsanopoulos, 1992; Kim and Yang, 2006), satellite image processing (Demirel et al., 2010), underwater image processing (Schechner and N. Karpel 2005; Singhai and Rawat, 2007), object tracking of video surveillance system (Chen et al., 2008), virtual restoration of ancient Chinese paintings (Pei et al., 2004), face recognition (Xie and Lam, 2005) and many more.

In a matter of fact, HE can be categorized into two major techniques which are the local HE (LHE) and the global HE (GHE) (Sengee et al., 2010). The GHE utilizes the whole histogram of the input image to compute a transformation function that will produce a stretched and flattened output image histogram. Thus, the contrast of the input image is enhanced (Kawakami et al., 2009). Although the GHE is well known for its simplicity and effectiveness in overall contrast enhancement but it cannot conserve the contrast and brightness of the original image due to the use of the whole histogram information. On the other hand, the LHE uses a sliding window method to generate a local intensities remapping for each pixel. Even if the LHE is good in improving the local contrast, it is time consuming and its implementation is complex. Therefore, in this work, only GHE is studied. Note that GHE will be referring as HE from now onward.

The basic idea of HE is to remap the gray levels of an image based on the image's gray levels cumulative density function. Mathematically, the probability density function of an input image, PDF is expressed as

$$PDF(k) = \frac{HIS(k)}{N} \quad \text{for } 0 \leq k \leq L-1 \quad (1.1)$$

where HIS is the histogram of the input image, L is the total number of gray levels (i.e., for an 8 bit image, $L = 256$), and N is total number of pixels and it can defined as

$$N = I_{width} \times I_{height} \quad (1.2)$$

where I_{width} and I_{height} are the image width and height respectively. Next, the cumulative density function of the input image, CDF is given as:

$$CDF(k) = \sum_{k=0}^{L-1} PDF(k) \quad \text{for } 0 \leq k \leq L-1 \quad (1.3)$$

Lastly, the transform function of the HE is expressed as:

$$T_{HE}(k) = l_0 + CDF(k) \times (l_{L-1} - l_0) \quad \text{for } 0 \leq k \leq L-1 \quad (1.4)$$

where l_0 and l_{L-1} are the gray level at the values of zero and $L-1$ respectively.

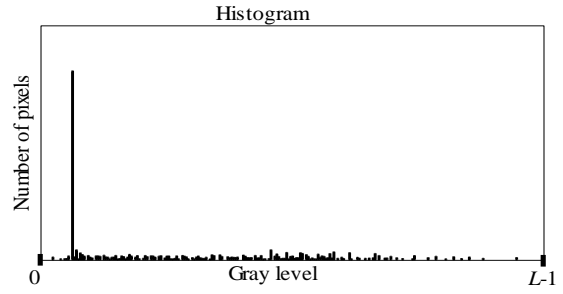
1.5 Problem Statements

Even though the HE is well known for its simplicity and effectiveness in image contrast enhancement, nevertheless, it suffers from excessive brightness change, intensity saturation and noise amplification problem (Chen and Manjit, 2009). This will degrade the quality of the enhanced images. By comparing the ‘Cat’ images in Figures 1.6 (a) and (c), it is obvious that there is an excessive brightness change at the background of the image in Figure 1.6 (c) (i.e., from black to gray color). Moreover due to the excessive brightness change, HE causes the shifting on the average (mean) luminance of the image (Kaur et al., 2011) which is well known as mean-shift problem (Kim and Chung 2008).

Besides, Figure 1.6 (c) also suffers from intensity saturation problem. This problem will cause undesirable detail loss. Intensity saturation problem occurs when large enhancement rate is applied to large bins’ region (i.e., in Figure 1.6 (b) gray level toward zero) while small bins’ region (i.e., in Figure 1.6 (b) gray level toward 255) are squeezed and merged into a bin, which will lead to information loss. Furthermore, HE also leads to noise amplification (i.e., background of the Figure 1.6(c)); this is due to high enhancement rate applied. Higher enhancement rate will lead to each detail becoming more enhanced, including the noise.



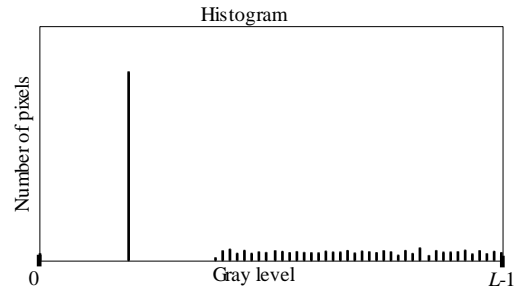
(a) 'Cat' image.



(b) Histogram of (a).



(c) HE-ed 'Cat' image.



(d) Histogram of (c).

Figure 1.6 Example of HE-ed image.

Moreover, for images with large area of background being black or white (i.e., 'Hand' image in Figure 1.7 (a) and Negative of 'Hand' in Figure 1.7 (c)), washed-out appearance (i.e., HE-ed 'Hand' image in Figure 1.7(b)) and patchiness effects (i.e., HE-ed negative of 'Hand' image in Figure 1.7(d)) occur after applying HE. The reason of the washed-out appearances after applying HE is that the input image has a large lower boundary value. From the HE transformation function, the larger the *PDF* of the lower boundary, the more the gray level will be shifted to the right. This will cause the washed-out appearances. On the other hand, if the input image has a large upper boundary value, the patchiness effects will occur after HE is applied.



(a) 'Hand' image.



(b) HE-ed 'Hand' image.



(c) Negative of 'Hand' image.



(d) HE-ed negative of 'Hand' image.

Figure 1.7 Example of washed-out appearances and patchiness effects (Chang and Chang, 2010).

Even though the HE method is widely used in many areas and applications, but in this work, the HE method being studied only focuses on improving the contrast of natural images. In short, the natural images being studied can be divided into two categories which are the low contrast images taken under normal lighting condition and the low contrast images taken under low lighting condition. The low contrast images taken under normal lighting condition require a contrast enhancement method that has the ability of brightness preservation, detail preservation and intensity saturation prevention. Meanwhile, for the low contrast images taken under low lighting condition, the brightness preservation is not needed, as these images appear dark. If brightness preservation is applied, the resultant image will remain dark. However, the detail preservation and intensity saturation prevention are needed while improving the contrast.

Based on the above explanation, the contrast enhancement method can be categorized into two categories (i.e., brightness preservation and detail preservation) which serve a different purpose of contrast enhancement. The brightness preservation methods had been developed to archive original mean brightness preservation and overcome the mean shift problem of the HE method. However, many of them tend to neglect the intensity saturation problem. These methods are mostly used to deal with low contrast images under normal lighting. On the other hand, the detail preservation methods are implemented to deal with low contrast images captured under low lighting condition by improving the visualization.

1.6 Objectives of Research

Based on the limitation of the HE as mentioned in Section 1.5, the main objectives of this work are as follow

- 1) To improve of the robustness of the existing HE based brightness preservation method namely, Quantized Plateau Limits Bi-Histogram Equalization method.
- 2) To implement a new HE based detail preservation method.

1.7 Scope of Research

In this research, the scope is focused on HE based contrast enhancement methods. The HE based methods can be categorized into two categories, which are brightness preservation and detail preservation methods. The former method is designed to deal with images captured under normal lighting condition, while the latter method is designed to deal with images captured under low lighting condition. On the other hand, the input images are limited to grayscale natural images only. These input images are separated into two categories, namely standard images taken

from internet sources and real images captured using personal camera (i.e., Canon EOS 50D). The real images can be further categorized into two categories, namely images captured under normal lighting condition and images captured under low lighting condition.

The comparison methods selected for brightness preservation method are the Histogram Equalization (HE), the Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE), the Simple Histogram Modification Scheme (SHMS), the Recursively Separated and Weighted Histogram Equalization (RSWHE), the Quantized Plateau Limits Bi-Histogram Equalization (QPLBHE), the Brightness Preserving Plateau Limit Histogram Equalization (BPPLHE), the Weight Clustering Histogram Equalization (WCHE) and the Weighting Mean-separated Sub-Histogram Equalization (WMSHE) methods. On the other hand, the selected comparison methods for the detail preservation method are the Histogram Equalization (HE), the Gain Controllable Clipped Histogram Equalization (GCCHE), the Weighted Threshold Histogram Equalization (WTHE), the Dynamic Histogram Equalization (DHE) and the Detail Preservation Plateau Limit Histogram Equalization (DPPLHE).

The development of the proposed algorithms and the comparison methods will be implemented by using the Matlab programming language. The inputs images are processed by using the processor of Pentium Dual CPU 4300 1.80GHz and 1.49GB of RAM.

1.8 Thesis Outline

In short, this thesis consists of 5 chapters which are Chapter 1 introduction, Chapter 2 Literature Reviews, Chapter 3 Methodology, Chapter 4 Results and Discussion and Chapter 5 Conclusion and Future Works.

Chapter 2 will discuss about the current existing HE based methods on contrast enhancement. The contrast enhancement is then separated into two categories namely brightness preservation and detail preservation methods. Each contrast enhancement category will be discussed in details. The discussion includes the fundamental concept, current works, advantages and disadvantages of these works. For comparison purpose, some of these methods will be implemented. Thus, verification of the implementation of the compared methods will be carried out at the end of Chapter 2.

Next, in Chapter 3, the methodology of the proposed brightness preservation method and detail preservation method is presented. The details (i.e., concept, advantages and disadvantages) of the proposed methods will be discussed. Next, image analysis and data sample to be used and implemented in this study will be discussed.

For Chapter 4, results and discussion of the performance of the proposed brightness preservation and detail preservation methods with the compared methods are presented. The enhanced images are compared based on the qualitative and quantitative analyses. Qualitative analysis will provide the visual inspection under good lighting condition, while the quantitative analysis numerically measures the brightness, contrast, entropy and runtime of the implemented methods.

Lastly, Chapter 5 concludes and summarizes the work done in this thesis. This chapter also includes the suggestions and future works of the work done.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

In general, contrast enhancement produces an output image which subjectively looks better and provides better representation of the subtle detail than the original image by changing the intensity values of the input image. It is an indispensable tool for researchers in a wide variety of fields including (but not limited to) medical imaging, color image processing and underwater image processing.

In short, Section 2.2 shows that the contrast enhancement method can be categorized into two categories (i.e., brightness preservation and detail preservation) which serve a different purpose of contrast enhancement. In Section 2.3, the brightness preservation methods will be discussed in details. These methods are mostly used to deal with low contrast images taken under normal lighting condition. The objective of these methods is to maintain the mean brightness while improving the contrast of the input images. Although the brightness preservation methods can be divided into many families, but only those families that are widely used which are the Bi-Histogram Equalization (BHE) based, the Recursive Histogram Equalization (RHE) based and the MHE Multiple Histogram Equalization (MHE) based methods are discussed.

On the other hand, the detail preservation methods are introduced to improve the visualization of the low contrast images taken under low lighting condition and will be discussed in detail in Section 2.4. The existing HE based detail preservation methods can be categorized into three families, which are the Plateau Limit

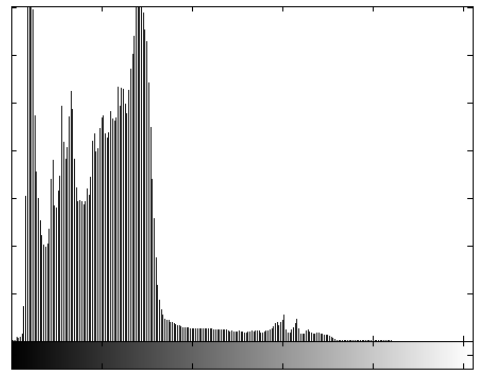
Histogram Equalization (PLHE) based, the Weighted Histogram Equalization (WHE) based and the Dynamic Range Histogram Equalization (DRHE) based methods. In Section 2.5, verification of implementation of the selected comparison methods will be discussed. Lastly, Section 2.6 will summarize the content of Chapter 2.

2.2 Contrast Enhancement Method

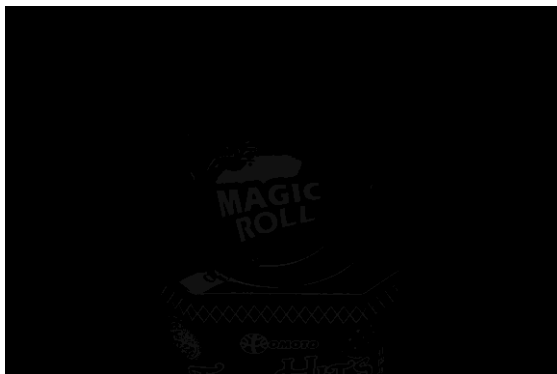
Basically, low contrast natural images can be divided into two types, namely images taken under normal lighting condition and images taken under low lighting condition. Examples of these images are as shown in Figure 2.1.



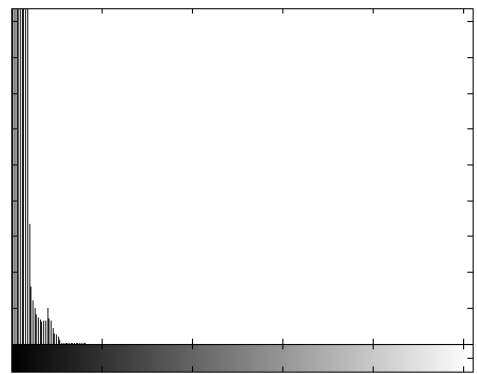
(a) An image taken under normal lighting condition.



(b) Histogram of (a).



(c) Image (a) taken under low lighting condition.



(d) Histogram of (c).

Figure 2.1 Example of image taken under normal lighting condition and image taken under low lighting condition.

Different type of input images requires different type of contrast enhancement method. Therefore, in order to deal with them, two categories of

contrast enhancement methods are introduced, namely brightness preservation and detail preservation methods. Brightness preservation method is designed to deal with images taken under normal condition by providing the ability of brightness preservation. The ability of brightness preservation which solves the mean shift problem is very useful in the consumer electronic products such as television and camera. On the other hand, the detail preservation method is implemented to improve the visual of the images taken under low lighting condition by increasing the mean brightness of the input images.

2.3 Brightness Preservation Method

The objective of brightness preservation method is to reduce the mean brightness shifting problem of the HE based method. The existing brightness preservation in HE based methods can be categorized into several groups, for example the Single Histogram Equalization (SHE) based, the Bi-Histogram Equalization (BHE) based, the Recursive-Histogram Equalization (RHE) based, the Multiple Histogram Equalization (MHE) based, the Fuzzy Histogram Equalization (FHE) based, the Object based Histogram Equalization (OBHE) based, the Neighborhood Metric Histogram Equalization (NMHE) based and other methods.

The feature of the SHE based method is that it has only one histogram. Examples of the SHE based methods are the Histogram Equalization with Variable Enhancement Degree (HEwVED) (Murahira et. al., 2010), the Simple Histogram Modification Scheme (SHMS) (Chang and Chang, 2010), and the Histogram Equalization with Range Offset (HERO) (Ibrahim, 2011). Next, the BHE based method has two sub-histograms. Several common techniques in this family are the Brightness Preserving Bi-Histogram Equalization (BBHE), (Kim, 1997), the

Dualistic Sub-Image Histogram Equalization (DSIHE) (Wang et. al., 1999) and the Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE) (Chen and Ramli, 2003b). The RHE based method has 2-based multiplication of sub-histograms. The RHE based methods include the Recursive Mean Separate Histogram Equalization (RMSHE) (Chen and Ramli, 2003a), the Recursive Sub-Image Histogram Equalization (RSIHE) (Sim et. al., 2007) and the Recursively Separated and Weighted Histogram Equalization (RSWHE) (Kim and Chung, 2008). Meanwhile, the MHE based method has several sub-histograms. Examples of techniques fall within this category are the Multipeak Histogram Equalization with Brightness Preservation (MHEBP) (Wongsritong et. al., 1998), the Brightness Preserving Dynamic Histogram Equalization (BPDHE) (Ibrahim and Kong, 2007) and the Weight Clustering Histogram Equalization (WCHE) (Sengee et. al., 2009).

On the other hand, the FHE based method is a hybrid technique of fuzzy and HE, for example, the Brightness Preserving Dynamic Fuzzy Histogram Equalization (BPDFHE) (Sheet et. al., 2010). Next, the OBHE based method extracts the foreground and background, before modifying and applying HE independently (Maragatham, 2011). Finally, the NMHE based method is a method that taking consideration of local and global information, for example, the Bi-Histogram Equalization with Neighborhood Metric (BHENM) (Sengee, 2010).

However, in this research, the brightness preservation methods are only focused on those families that are widely used which are the BHE based, the RHE based and the MHE based methods. These families will be discussed in details in the next sub-sections.

2.3.1 Bi-Histogram Equalization (BHE) Based Brightness Preservation Method

Generally, the BHE based methods have one separating point, SP , which divides the histogram into two sub-histograms, namely lower sub-histogram, HIS_L and higher sub-histogram, HIS_H as illustrated in Figure 2.2. Normally, the SP is set equal to either mean or median of the input histogram. Sometimes, the setting procedure is user defined or is selected based on certain criterion or equation.

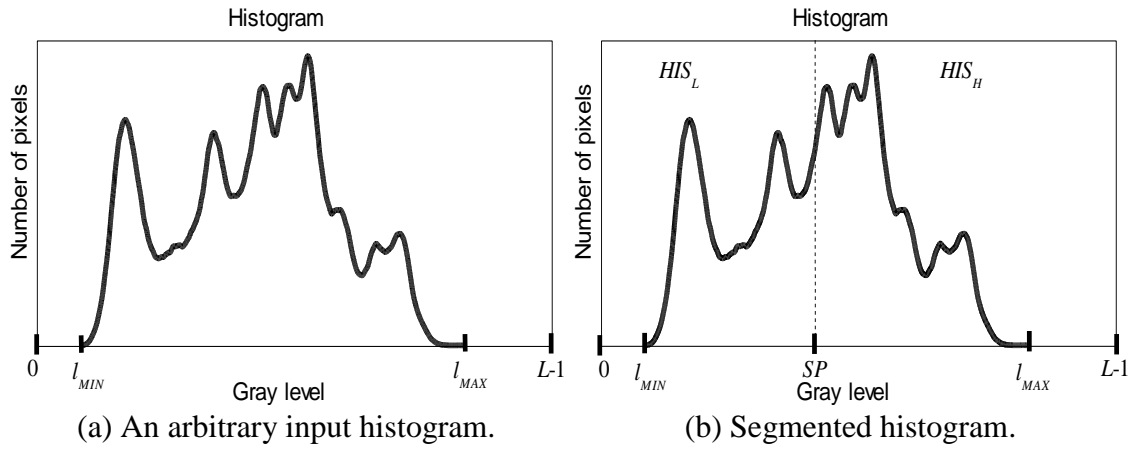


Figure 2.2 The procedure of the BHE based histogram segmentation.

The HIS_L contains the pixels from minimum gray level, l_{MIN} to SP . Meanwhile, the HIS_H contains the pixels from $SP + 1$ to maximum gray level, l_{MAX} . Finally, HE is separately applied to each sub-histogram directly or after the sub-histogram modification if there is.

2.3.1 (i) Brightness Preserving Bi-Histogram Equalization (BBHE)

The Brightness Preserving Bi-Histogram Equalization (BBHE) is introduced by Kim (1997) to preserve the mean brightness of an image. The SP for BBHE, SP_{BBHE} is determined based on the mean of the histogram, which is defined as

$$SP_{BBHE} = \frac{\sum_{k=0}^{L-1} k \times n_k}{N} \quad (2.1)$$

where n_k is the bins of the k -th gray level. Then, the SP is used to separate the histogram of the input image into the lower and higher sub-histograms. After that, the HE is applied to each sub-histogram separately. As the HE is applied on each sub-histogram separately, the mean brightness of the input image could be maintained. However, according to Wang and Ye (2005), the BBHE can be only worked well for images with histogram that have symmetrical distribution around its mean. Otherwise, the BBHE cannot preserve the mean brightness.

2.3.1 (ii) Dualistic Sub-Image Histogram Equalization (DSIHE)

Next, the Dualistic Sub-Image Histogram Equalization (DSIHE) was introduced to preserve mean-brightness and image detail (Wang et. al., 1999). The concept of the DSIHE and BBHE is similar, except the selection of the SP value. The SP value for DSIHE is selected based on the median of the histogram. The authors have proved that this would yield maximum entropy for the enhanced image. The SP for DSIHE, SP_{DSIHE} can be calculated as

$$SP_{DSIHE} = l_k, \quad \text{if } CDF(l_k) = 0.5 \quad (2.2)$$

where l_k is the k -th gray level. Nevertheless, there are still cases that are not well handled. The images with a large uniform background will lead the DSIHE to produce enhanced images with unnatural contrast enhancement and annoying artifacts (Chen and Ramli, 2003).

2.3.1 (iii) Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE)

Later, the Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE) was proposed to provide maximum brightness preservation (Chen and Ramli, 2003b). The selected SP is the SP which would yield minimum Absolute Mean Brightness Error ($AMBE$). The $AMBE$ is the absolute mean difference between the input image and enhanced image. It is given as

$$AMBE = |E(Y) - E(X)| \quad (2.3)$$

where $E(Y)$ and $E(X)$ are the mean of the enhanced and the input image respectively. To determine the optimum value of SP , firstly, each gray level is temporary set as the SP and the $AMBE$ value is calculated for each SP . For each iteration, a complete BBHE process is applied. The SP with minimum $AMBE$ value is selected as the final SP for the MMBEBHE. Then, the final SP value is used to separate the histogram of the input image into two sub-histograms. Finally, the HE is applied to each sub-histogram separately for contrast enhancement purpose. Finding the optimum SP value would require considerable amount of computation, so in order to facilitate in real time implementation, an efficient recursive integer-based computation for the $AMBE$ has been introduced.

The estimate output mean with separating point set as K can be defined as

$$E_K(Y) = \left(\frac{l_0 + K}{2}\right) \left[\sum_{i=0}^K PDF(l_i)\right] + \left(\frac{K+1+l_{L-1}}{2}\right) \left[\sum_{i=K+1}^{L-1} PDF(l_i)\right] \quad (2.4)$$

After simplification, the simplified estimate mean is represented by

$$E_K(Y) = \frac{1}{2} [K + L(1 - \sum_{i=0}^K PDF(l_i))] \quad (2.5)$$

Note that, during the simplification, l_0 is assumed equal to zero (this assumption can be corrected automatically in the later part) and $1 + l_{L-1}$ is equal to L .

Next, the estimate output mean with separating point set as $K + 1$ can be defined as

$$\begin{aligned} E_{K+1}(Y) &= \frac{1}{2} [K + 1 + L(1 - \sum_{i=0}^{K+1} PDF(l_i))] \\ &= \frac{1}{2} [K + 1 + L(1 - \sum_{i=0}^K PDF(l_i) - PDF(K + 1))] \\ &= \frac{1}{2} [K + L(1 - \sum_{i=0}^K PDF(l_i))] + \frac{1}{2} [1 - L \times PDF(K + 1)] \\ &= E_K(Y) + \frac{1}{2} [1 - L \times PDF(K + 1)] \end{aligned} \quad (2.6)$$

From equations 2.5 and 2.6, it can be observed that the estimate output mean has a recursion relationship. So the estimate output mean can be calculated as

$$\begin{aligned} E_0(Y) &= \frac{1}{2} [L(1 - PDF(l_0))] \\ E_1(Y) &= E_0(Y) + \frac{1}{2} [1 - L \times PDF(l_1)] \\ &\dots\dots \\ E_K(Y) &= E_{K-1}(Y) + \frac{1}{2} [1 - L \times PDF(K)] \end{aligned} \quad (2.7)$$

After that, the input mean with similar assumption ($l_0 = 0$) can be computed as

$$E(X) = \sum_{i=0}^{L-1} i \times PDF(l_i) \quad (2.8)$$

Note that during the calculation of Mean Brightness Error (*MBE*) value, the assumption ($l_0 = 0$) can be corrected automatically. If $l_0 = 0$ is not true, correction can be done by adding the actual l_0 to both $E(X)$ and $E(Y)$. Then, these two terms will cancel off each other in the calculation of *MBE* as

$$MBE = (E(Y) + l_0) - (E(X) + l_0) = E(Y) - E(X) \quad (2.9)$$

Therefore, the *MBE* for each separating point can be defined as

$$MBE_0 = E_0(Y) - E(X) = \frac{1}{2}[L(1 - PDF(l_0))] - E(X)$$

$$MBE_1 = E_1(Y) - E(X)$$

$$= E_0(Y) + \frac{1}{2}[1 - L \times PDF(l_1)] - E(X)$$

$$= MBE_0 + \frac{1}{2}[1 - L \times PDF(l_1)]$$

.....

$$MBE_K = MBE_{K-1} + \frac{1}{2}[1 - L \times PDF(K)] \quad (2.10)$$

Note that the MMBEBHE only requires the separating point with the smallest *AMBE* value, so it does not really matter if the *MBE* is a floating point or integer number. Since floating point number requires longer processing time, thus the *MBE* should be converting to integer number to fasten the process. From equation 2.10, the *MBE* is clearly a floating point number, as the *PDF* value is in a floating point number. Therefore, a scaled *MBE* (*SMBE*) with integer number should be sufficient. In order to change *MBE* to scaled *MBE*, a scale factor is needed. So, to obtain the scale factor, the *MBE* from equation 2.10 should be changed to equation 2.11.

$$MBE_0 = \frac{1}{2}[L(1 - PDF(l_0))] - E(X)$$

$$\begin{aligned}
&= \frac{1}{2} \left[L \left(1 - \frac{HIS(l_0)}{N} \right) \right] - \sum_{i=0}^{L-1} \frac{i \times HIS(l_i)}{N} \\
&= \frac{1}{2N} \left[L(N - HIS(l_0)) - 2 \sum_{i=0}^{L-1} i \times HIS(l_i) \right] \tag{2.11}
\end{aligned}$$

From equation 2.11, the scale factor is clearly seen as $2N$. Therefore, the $SMBE$ can be defined as

$$\begin{aligned}
SMBE_0 &= (2N)MBE_0 \\
&= L(N - HIS(l_0)) - 2 \sum_{i=0}^{L-1} i \times HIS(l_i) \\
SMBE_1 &= (2N)MBE_1 \\
&= (2N) \left[MBE_0 + \frac{1}{2} (1 - L \times PDF(l_1)) \right] \\
&= (2N) \left[MBE_0 + \frac{1}{2N} (N - L \times HIS(l_1)) \right] \\
&= (2N)MBE_0 + (N - L \times HIS(l_1)) \\
&= SMBE_0 + (N - L \times HIS(l_1))
\end{aligned}$$

.....

$$SMBE_K = SMBE_{K-1} + (N - L \times HIS(K)) \tag{2.12}$$

Since the number of gray level is always of base 2, so the L could be replaced with basic shift operation. Suppose $L = 2^z$, then the $SMBE$ is as in equation 2.13. To find the absolute value of $SMBE$, a comparator is used.

$$\begin{aligned}
SMBE_0 &= [(N - HIS(l_0)) << z] - \left(\sum_{i=0}^{L-1} i \times HIS(l_i) \right) << z \\
SMBE_1 &= SMBE_0 + (N - (HIS(l_1)) << z)
\end{aligned}$$

.....

$$SMBE_K = SMBE_{K-1} + (N - (HIS(K)) \ll z) \quad (2.13)$$

However, due to the highly complex implementation, the MMBEBHE requires longer runtime. Moreover, the minimal mean shift in the output does not always guarantee the visual quality of the enhanced image.

2.3.1 (iv) Simple Histogram Modification Scheme (SHMS)

On the other hand, the Simple Histogram Modification Scheme (SHMS) is implemented to remove the washed-out appearances and the patchiness effect after applying HE (Chang and Chang, 2010).

For the implementation of the SHMS, the first non-zero bin is firstly found from the input histogram and is then replaced to zero bin. Next, the last two non-zero bins are found before the last non-zero bin is replaced with the minimum bin between the last two non-zero bins. Finally, HE is applied to the modified histogram. This modification is intended to solve the washed-out appearance and patchiness effect.

According to Chang and Chang (2010), the SHMS can be used for both single and multiple thresholding cases. The main advantage is that the SHMS can solve the washed-out appearance and patchiness effect. However, for the case of multiple thresholds, it limits brightness to certain extent. As a result, higher degree of brightness preservation is required to avoid generating annoying artefacts in the enhanced images.